**Attendance Problems.**

1. Sketch a right angle and its angle bisector.

2. Draw three different squares with (3, 2) as one vertex.

3. Find the values of $x$ and $y$ if $(3, -2) = (x + 1, y - 3)$

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- I can identify reflections, rotations, and translations.
- I can graph transformations in the coordinate plane.

**Common Core**

- **CC.9-12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- **CC.9-12.G.CO.2** Represent transformations in the plane using.
- **CC.9-12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure...Specify a sequence of transformations that will carry a given figure onto another.
4. Consider what you know about rotations, a motion that turns a shape about a point. Does it make any difference if a rotation is clockwise versus counterclockwise? If so, when does it matter? Are there any circumstances when it does not matter? And are there any situations when the rotated image lies exactly on the original shape?

Investigate these questions as you rotate the shapes about the given point. Using tracing paper if needed. Be prepared your answers to the questions posed.

- a. $180^\circ$
- b. $180^\circ$
- c. $90^\circ$
- d. $90^\circ$
- e. $270^\circ$
- f. $360^\circ$
- g. $180^\circ$
- h. $90^\circ$

So what exactly is a rotation? If a figure is rotated, how can you describe it? Investigate this question.

5. Open up sketchpad and graph $\overline{AB}$ with coordinates A(4, 1) and B(2, 5). Mark the origin as the center of rotation. Name the origin point O. Do this by double-clicking on the origin. Select $\overline{AB}$ and under the transform menu, choose rotate by a fixed angle. Make the angle $90^\circ$. What are the coordinates of A’ & B’?
6. Measure the length of \( \overline{AB} \) & \( \overline{A'B'} \). Measure the distance from O to A, O to A’, O to B, O to B’. How do they compare?

7. Measure \( \angle AOA' \) & \( \angle BOB' \). How do the angles compare?

8. Why does it make sense that for all points P in the graph, a rotation about a point O moves it to a new point P’ so that \( OP = OP' \) and \( m \angle POP' \) equals the measure of rotation?

9. Explain why a rotation does not change an angles or lengths of a figure. (Hint: Refer to question 4.)

So what is a translation? The formal name for a slide is a translation. (Remember that translation and transformation are different words) \( \triangle A'B'C' \) at is the result of translating \( \triangle ABC \).

10. Describe the translation. That is, how many units to the right and how many units down does the translation move the triangle?
11. On graph paper, plot \( \triangle EFG \) with coordinates \( E(4, 2), F(1, 7), \) and \( G(2, 0) \). Find the coordinates of \( \triangle E'F'G' \) if \( \triangle E'F'G' \) is translated the same way as \( \triangle ABC \) was in problem 10.

12. For the translated triangle in problem 10, draw a line segment connecting each vertex to its translated image. What do you notice about these line segments? What does this tell you about how a translation moves each point on the graph?

14. Explain how you know that a translation does not change angles or lengths of a figure. If necessary, use tracing paper.
How can transformations such as reflections help us to learn more about familiar shapes. Consider reflecting a line segment across a line that passes through one of its endpoints. An example of this would be reflecting $\overline{AB}$ across $\overline{BC}$.

15. Go to the class website (http://watertowngeometry.wikispaces.com), click on unit 1 (http://watertowngeometry.wikispaces.com/Unit+1+Notes) and open the sketchpad document, isosceles triangle. Click on $\overline{BC}$ and under the transform menu, click on mark “Mark Mirror.” Alternatively, you can double click on $\overline{BC}$. Click on $\overline{AB}$ and reflect it across $\overline{BC}$. When points A, B, & A’ are connected, what figure is formed?

16. Use what you know about reflection to make as many statements as you about the shape from problem 15. For example, are there any sides that must be the same length? Are there any angles that must be equal? Is there anything else special about this shape? Click and drag some points to test your conjectures.

**Formal Definition**

In algebra, you learned that a function is an equation that assigns each input a unique output. Most of these functions involved expressions and numbers such as $f(x) = 3x - 5$, so $f(2) = 1$.

In this course, you are now studying functions that assign each point in the the plane to a unique a point in the plane. These functions are called **rigid transformations (or isometries)** because they move the entire plane with any figures your have drawn so that all the figures remain unchanged. Therefore, angles and distances are preserved. There three basic isometries that we will consider: reflections, translations, and rotations. All rigid motions can be seen as a combination of them.

**Reflections:** When a figure is reflected across a line of reflections, such as the shape shown, it appears that the figure is “flipped” over the line. However, formally, a reflection across a line of reflection is defined as a function of each point (such as A) to point (such as A’) so that the line of reflection is the perpendicular bisector of the segments connecting the points and their images (such as $\overline{AA'}$). Therefore, $AP = A'P$.

**Rotation:** Formally, a rotation about point O is function that assigns
each point (P) in the plane a unique point (P’) so that all angles of rotation $\angle POP'$ have the same measure (which is the angle of rotation) and $OP = OP'$.

**Translation:** Formally, a translation is a function that assigns each point (Q) in the plane to a unique point (Q’) so that all line segments connecting an original point with its image have equal lengths and are parallel.
17. **Isosceles triangle.** When two sides of a triangle have the same length, that triangle is called **isosceles**. Describe all the facts you know about isosceles triangles based on the reflection. Be sure to include a diagram.
Video Example 1. Identify the transformation. Then use arrow notation to describe the transformation.

A.

B.

Identifying Transformations

Identify the transformation. Then use arrow notation to describe the transformation.

A. The transformation cannot be a translation because each point and its image are not in the same position.

The transformation is a reflection. \( \triangle EFG \rightarrow \triangle E'FG' \)
Identify the transformation. Then use arrow notation to describe the transformation.

The transformation cannot be a reflection because each point and its image are not the same distance from a line of reflection.

The transformation is a 90° rotation. $RSTU \rightarrow R'S'T'U'$

Example 1. Identify the transformation. Then use arrow notation to describe the transformation.

A.

B.

To find coordinates for the image of a figure in a translation, add $a$ to the $x$-coordinates of the preimage and add $b$ to the $y$-coordinates of the preimage.

Translations can also be described by a rule such as $(x, y) \rightarrow (x+a, y+b)$.

Example 2. Translations in the Coordinate Plane

Find the coordinates for the image of $\triangle ABC$ after the translation $(x, y) \rightarrow (x+3, y-4)$.
Guided Practice. Identify the transformation. Then use arrow notation to describe the transformation.

9. [Diagram of MNP with MNP' labeled]

10. [Diagram of XYZ with X'Y'Z' labeled]

Video Example 2. A figure has vertices at A(2, 4), B(4, -1), and C(2, -1). After a transformation, the image of the figure has vertices at A'(-2, 4), B'(-4, -1), and C'(-2, -1). Draw the preimage and the image. Then identify the transformation.

Drawing and Identifying Transformations

A figure has vertices at A(-1, 4), B(-1, 1), and C(3, 1). After a transformation, the image of the figure has vertices at A'(-1, -4), B'(-1, -1), and C'(3, -1). Draw the preimage and image. Then identify the transformation.

Plot the points. Then use a straightedge to connect the vertices.

The transformation is a reflection across the x-axis because each point and its image are the same distance from the x-axis.
**Example 2.** A figure has vertices at $A(1, -1)$, $B(2, 3)$, and $C(4, -2)$. After a transformation, the image of the figure has vertices at $A'(-1, -1)$, $B'(2, 3)$, and $C'(-4, -2)$. Draw the preimage and image. Then identify the transformation.

**11. Guided Practice:** A figure has vertices at $E(2, 0)$, $F(2, -1)$, $G(5, -1)$, and $H(5, 0)$. After a transformation, the image of the figure has vertices at $E'(0, 2)$, $F'(1, 2)$, $G'(1, 5)$, and $H'(0, 5)$. Draw the preimage and image. Then identify the transformation.

**Video Example 3.** Find the coordinates for the image of $\triangle ABC$ after the translation $(x, y) \rightarrow (x - 3, y + 2)$. Draw the image.
Translating in the Coordinate Plane

Find the coordinates for the image of $\triangle ABC$ after the translation $(x, y) \rightarrow (x + 3, y - 4)$. Draw the image.

**Step 1** Find the coordinates of $\triangle ABC$.
The vertices of $\triangle ABC$ are $A(1, 1)$, $B(3, 3)$, and $C(-4, 0)$.

**Step 2** Apply the rule to find the vertices of the image.

$A'(1 + 3, 1 - 4) = A'(2, -3)$

$B'(3 + 3, 3 - 4) = B'(0, -1)$

$C'(4 + 3, 0 - 4) = C'(1, -4)$

**Step 3** Plot the points. Then finish drawing the image by using a straightedge to connect the vertices.

**Example 3.** Find the coordinates for the image of $\triangle ABC$ after the translation $(x, y) \rightarrow (x + 2, y - 1)$. Draw the image.
12. Guided Practice: Find the coordinates for the image of JKLM after the translation $(x,y) \rightarrow (x-2, y+4)$. Draw the image.

**Video Example 4.** The pattern shown is similar to a pattern on a wall of the Alhambra. Write a rule for the translation of square to square 2.

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**Art History Application**

The pattern shown is similar to a pattern on a wall of the Alhambra. Write a rule for the translation of square 1 to square 2.

**Step 1** Choose 2 points
- Choose a point $A$ on the preimage and a corresponding point $A'$ on the image. $A$ has coordinates $(3, 1)$, and $A'$ has coordinates $(1, 3)$.

**Step 2** Translate
- To translate $A$ to $A'$, 2 units are subtracted from the $x$-coordinate and 2 units are added to the $y$-coordinate. Therefore, the translation rule is $(x, y) \rightarrow (x-2, y+2)$. 

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**Example 4.** The figure shows part of a tile floor. Write a rule for the translation of hexagon 1 to hexagon 2.

**10. Guided Practice:** Use the diagram to write a rule for the translation of square 1 to square 3.